

**SELF-INTERACTION FOR PARTICLES IN THE WORMHOLE
SPACE-TIMES**

NAIL KHUSNUTDINOV

*Institute of Physics, Kazan Federal University, Kremlevskaya 18
Kazan, 420008, Russia
7nail7@gmail.com*

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The self-energy and self-force for particles with electric and scalar charges at rest in the space-time of massless and massive wormholes are considered. The particle with electric charge is always attracted to wormhole throat for arbitrary profile of the throat. The self-force for scalar particle shows different behavior depending on the non-minimal coupling. The self-force for massive scalar field is localized close to the throat of the wormhole.

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1. Introduction

Wormholes are topological handles linking different regions of the Universe or different universes. The activity in wormhole physics was first initiated by the classical paper by Einstein and Rosen,¹ and later by Wheeler.² The latest growth of interest in wormholes was connected with the “time machine”, introduced by Morris, Thorne and Yurtsever in Refs. 3, 4. Their works led to a surge of activity in wormhole physics.⁵ The main and unsolved problem in wormhole physics is whether wormholes exist or not. The wormhole has to violate energy conditions and the source of the wormhole geometry should be exotic matter. One example^{6,7,8} of such exotic matter is quantum fluctuations, which may violate the energy conditions. Another possible sources are a scalar field with reversed sign of kinetic term,⁹ and cosmic phantom energy.^{10,11} The question of wormhole’s stability is not simple and requires subtle calculations (see, for example¹²). The problem arises because a wormhole needs some amount of exotic matter which violates energy conditions and has unusual properties. Recently it was observed that some observational features of black holes can be closely mimicked by spherically symmetric static wormholes¹³ having no event horizon. Some astronomical observations indicate possible existence of black holes (see, for example, Ref. 14). It is therefore important to consider possible astronomical evidences of the wormholes. Some aspects of wormholes’ astrophysics were considered in Ref. 15, where it was noted that this massive compact object

may correspond to a wormhole of macroscopic size with strong magnetic field. A matter may go in and come back out of the wormhole's throat.

In the framework of general relativity there exists a specific interaction of particles with gravitating objects – the gravitationally induced self-interaction force which may have considerable effect on the wormholes' physics. It is well-known¹⁶ that in a curved background alongside with the standard Abraham-Lorentz-Dirac self-force there exists a specific force acting on a charged particle. This force is the manifestation of non-local essence of the electromagnetic field. It was considered in details in some specific space-times (see Refs. 17, 18 for review). For example, in the case of the straight cosmic string space-time the self-force appears to be the only form of interaction between the particle and the string. Cosmic string has no Newtonian potential but nevertheless a massless charged particle is repelled by the string,¹⁹ whereas massive uncharged particle is attracted by the string due to the self-force.²⁰ The non-trivial internal structure of the string does not change this conclusion.²¹ The potential barrier appears which prevents the charged particle from penetrating into the string. For GUT cosmic strings the potential barrier is $\sim 10^5 \text{ GeV}$. The wormhole is an example of the space-time with non-trivial topology. The consideration of the Casimir effect for a sphere that surrounds the wormhole's throat demonstrates an unusual behavior of the Casimir force²² – it may change its sign depending on the radius of the sphere. It is expected that the self-force in the wormhole space-time will show an unusual behaviour too.

In this paper we review the self-interaction force for particle with electric and scalar charges in different kind of wormholes space-times.

2. The Massless Wormhole Space-Time

Let us consider an asymptotically flat wormhole space-time. We choose the line element of this space-time in the following form

$$ds^2 = -dt^2 + d\rho^2 + r^2(\rho)(d\theta^2 + \sin^2\theta d\varphi^2), \quad (1)$$

where $t, \rho \in \mathbf{R}$ and $\theta \in [0, \pi]$, $\varphi \in [0, 2\pi]$. Profile of the wormhole throat is described by the function $r(\rho)$. This space-time firstly was considered by Bronnikov²³ and Ellis.²⁴ This function must have a minimum at $\rho = 0$, and the minimal value at $\rho = 0$ corresponds to the radius, a , of the wormhole throat,

$$r(0) = a, \quad \dot{r}(0) = 0,$$

where an over dot denotes the derivative with respect to the radial coordinate ρ . Space-time is naturally divided into two parts in accordance with the sign of ρ . We shall label the part of the space-time with positive (negative) ρ and the functions on this part with the sign "+" ("−").

The space-time possesses non-zero curvature. The scalar curvature is given by

$$R = -\frac{2(2r\ddot{r} + \dot{r}^2 - 1)}{r^2}.$$

Far from the wormhole throat the space-time becomes Minkowskian,

$$r(\rho)|_{\rho \rightarrow \pm\infty} = \pm\rho. \quad (2)$$

Various kinds of throat profiles have been already considered in another context.⁷ The simplest model of a wormhole is that with an infinitely short throat,⁷

$$r = a + |\rho|. \quad (3)$$

The space-time is flat everywhere except for the throat, $\rho = 0$, where the curvature has delta-like form,

$$R = -8 \frac{\delta(\rho)}{a}.$$

Another wormhole space-time that is characterized by the throat profile

$$r = \sqrt{a^2 + \rho^2} \quad (4)$$

is free of curvature singularities:

$$R = -\frac{2a^2}{(a^2 + \rho^2)^2}.$$

The wormholes with the following profiles of throat

$$\begin{aligned} r &= \rho \coth \frac{\rho}{b} + a - b, \\ r &= \rho \tanh \frac{\rho}{b} + a, \end{aligned}$$

have a throat whose length may be described using a parameter b . The point is that for $\rho > b$ the space-time becomes Minkowskian exponentially fast.

3. Self-Energy and Self-Force

The self-energy of particle at rest with electric charge e is defined as one-half of coincidence limit of the renormalized interaction energy of particle with the same charge. For particle at rest with scalar charge the expression for self-energy was found in Ref. 25 and it has the same form as for electric case

$$U = \frac{e^2}{2} G^{ren}(x, x). \quad (5)$$

Let us consider²⁶ a charged particle at rest in the point ρ', θ', φ' in the space-time with metric (1). The Maxwell equation for zero component of the potential reads

$$\Delta A^0 = -\frac{4\pi e \delta(\rho - \rho')}{r^2(\rho)} \frac{\delta(\theta - \theta') \delta(\varphi - \varphi')}{\sin \theta}$$

where $\Delta = g^{kl} \nabla_k \nabla_l$. Due to static character of the background we set other components of the vector potential to be zero. It is obvious that $A^0 = 4\pi e G(\mathbf{x}; \mathbf{x}')$, where the three-dimensional Green's function G obeys the following equation

$$\Delta G(\mathbf{x}; \mathbf{x}') = -\frac{\delta(\rho - \rho')}{r^2(\rho)} \frac{\delta(\theta - \theta') \delta(\varphi - \varphi')}{\sin \theta}.$$

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Due to spherical symmetry we may extract the angular dependence (denoting succinctly $\Omega = (\theta, \varphi)$)

$$G(\mathbf{x}; \mathbf{x}') = \sum_{l=0}^{\infty} \sum_{m=-l}^l Y_{lm}(\Omega) Y_{lm}^*(\Omega') g_l(\rho, \rho'),$$

and introduce the radial Green's function g_l subject to the equation

$$\ddot{g}_l + \frac{2r'}{r} \dot{g}_l - \frac{l(l+1)}{r^2} g_l = -\frac{\delta(\rho - \rho')}{r^2}. \quad (6)$$

We represent the solution of this equation in the following form

$$g_l = \theta(\rho - \rho') \Psi_1(\rho) \Psi_2(\rho') + \theta(\rho' - \rho) \Psi_1(\rho') \Psi_2(\rho), \quad (7)$$

where functions Ψ are the solutions of the corresponding homogeneous equation

$$\ddot{\Psi} + \frac{2r'}{r} \dot{\Psi} - \frac{l(l+1)}{r^2} \Psi = 0, \quad (8)$$

satisfying the boundary conditions

$$\lim_{\rho \rightarrow +\infty} \Psi_1 = 0, \quad \lim_{\rho \rightarrow +\infty} \Psi_2 = \infty. \quad (9)$$

If one substitutes (7) to (6) the condition for the Wronskian emerges:

$$W(\Psi_1, \Psi_2) = \Psi_1 \dot{\Psi}_2 - \dot{\Psi}_1 \Psi_2 = \frac{1}{r^2(\rho)}. \quad (10)$$

In the case of scalar massive particle with scalar field with non-minimal coupling we have different radial equation

$$g_l'' + \frac{2r'}{r} g_l' - \left(m^2 + \frac{l(l+1)}{r^2} + \xi R \right) g_l = -\frac{\delta(\rho - \rho')}{r^2}. \quad (11)$$

We consider the radial equation in domains $\rho > 0$ and $\rho < 0$ and obtain a pair of independent solutions ϕ^1, ϕ^2 for each of the domains separately. We do not need to consider two domains if it is possible to construct solutions that are C^1 -smooth over all space. However, this is not the case for many situations. For the two kinds of the throat profile considered below we may easily construct solutions for the two domains separately (but not for all space). After that a procedure developed here allows to construct C^1 -smooth solution over all space.

Let us consider in detail the simple case of the symmetric throat profile: $r(-\rho) = r(\rho)$. In this case we obtain the radial Green function

1. $\rho > \rho' > 0$

$$g_l^{(1)}(\rho, \rho') = -\frac{1}{A_+} \phi_+^2(\rho') \phi_+^1(\rho) + \frac{1}{A_+} \frac{W_+(\phi_+^1, \phi_+^2)}{W_+(\phi_+^2, \phi_+^2)} \Big|_0 \phi_+^2(\rho') \phi_+^2(\rho) \quad (12a)$$

2. $\rho < \rho'$ and $\rho' > 0, \rho < 0$

$$g_l^{(2)}(\rho, \rho') = -\frac{1}{A_+} \frac{W_+(\phi_+^1, \phi_+^2)}{W_+(\phi_+^2, \phi_+^2)} \Big|_0 \phi_+^2(\rho') \phi_+^2(-\rho) \quad (12b)$$

Here $W_+(y_1, y_2) = y_1 \dot{y}_2 - \dot{y}_1 y_2$ and $A_{\pm} = W(\phi_{\pm}^1, \phi_{\pm}^2) r^2(\rho)$.

3.1. Profile $r = a + |\rho|$

i) Electromagnetic field. The Green function reads²⁶

$$4\pi G^{(1)}(x; x') = \frac{1}{\sqrt{r(\rho)^2 - 2r(\rho)r(\rho')\cos\gamma + r(\rho')^2}} - \frac{1}{2a} \ln \left| 1 + \frac{2t}{1-t + \sqrt{t^2 - 2t\cos\gamma + 1}} \right|,$$

$$4\pi G^{(3)}(x; x') = \frac{t}{a\sqrt{t^2 - 2t\cos\gamma + 1}} - \frac{1}{2a} \ln \left| 1 + \frac{2t}{1-t + \sqrt{t^2 - 2t\cos\gamma + 1}} \right|,$$

where $t = \frac{a^2}{r(\rho)r(\rho')}$. The surfaces of constant energy are shown in Fig. 1.

The self-potential is

$$U = \frac{e^2}{4a} \ln \left[1 - \frac{a^2}{(a + \rho)^2} \right]. \quad (13)$$

The self-force

$$\mathbf{F} = -\nabla U$$

has the radial component only

$$F^\rho = -\partial_\rho U = -\frac{ae^2}{2r(\rho)^3} \frac{1}{1 - \frac{a^2}{r(\rho)^2}}.$$

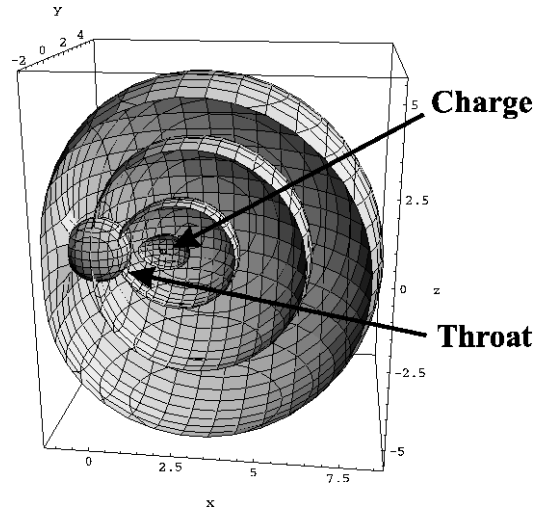


Fig. 1. Several surfaces of constant potential are shown. The charge $e = 1$ is at the point $x = 2, y = z = 0$. The small sphere is the throat of the wormhole, $\rho = 0$. We observe that some surfaces go under the throat to another universe.

The self-force is always attractive, it turns into infinity at the throat and goes down monotonically to zero as $\rho \rightarrow \infty$. We may compare this expression with its analog for Schwarzschild space-time with Schwarzschild radius $r_s = a$:

$$F^r = +\frac{ae^2}{2r^3}\sqrt{1 - \frac{a^2}{r^2}}.$$

Important observations are:

- 1) The self-force in the wormhole space-time has an opposite sign – it is attractive.
- 2) Far from the wormhole throat and from the black hole we have the same results but with opposite signs

$$\begin{aligned} F_{wh}^\rho &= -\frac{ae^2}{2\rho^3}, \\ F_{bh}^\rho &= +\frac{ae^2}{2\rho^3}. \end{aligned}$$

3) At the Schwarzschild radius $r_s = a$ the self-force equals zero, whereas at the wormhole throat it tends to infinity. The latter discrepancy originates in the selected throat profile function that leads to the curvature singularity at the throat.

ii) Scalar field. For massless particle the self-potential read²⁵

$$U(\rho) = -\frac{ae^2(1-8\xi)}{4r^2}\Phi\left(\frac{a^2}{r^2}, 1, 1-4\xi\right), \quad (14)$$

where

$$\Phi\left(\frac{a^2}{r^2}, 1, 1-4\xi\right) = \sum_{n=0}^{\infty} (1-4\xi+n)^{-1} \left(\frac{a}{r}\right)^{2n}. \quad (15)$$

For massive particle we obtain for self-energy the following expression

$$U(\rho) = -e^2 \sum_{l=0}^{\infty} \nu \frac{ma(I_\nu K'_\nu + I'_\nu K_\nu) + (8\xi-1)I_\nu K_\nu}{2maK_\nu K'_\nu + (8\xi-1)K_\nu^2} \Big|_{ma} \frac{K_\nu^2(mr)}{r}, \quad (16)$$

where I_ν and K_ν are the Bessel function of the second kind.

The numerical simulations of the self-energy are shown in Fig. 2. We note that the massive field will produce a self-force which is localized close to the throat. It falls down exponentially fast as e^{-mr} far from the throat. This behavior is in agreement with Linet result.²⁷

3.2. Profile $r = \sqrt{a^2 + \rho^2}$

i) Electromagnetic field. The self-energy and self-force read²⁶

$$U = -\frac{e^2}{2\pi} \frac{a}{\rho^2 + a^2}, \quad F^\rho = \partial_\rho U = -\frac{e^2}{\pi} \frac{a\rho}{(\rho^2 + a^2)^2}. \quad (17)$$

The self-force is everywhere finite and equals zero at the throat. Far from the wormhole we have

$$F^\rho \approx -\frac{e^2}{\pi} \frac{a}{\rho^3}.$$

Thus the self-force is always attractive. It has maximum value at distance $\rho^* = a/\sqrt{3}$ with magnitude $F_{max}^\rho = 3\sqrt{3}e^2/16\pi a^2$. The plots of the potential and the self-force are shown in the Fig. 3.

ii) Scalar field. Massless case. The expression for this case was obtained in Ref. 25 and it has the following form ($\mu = \sqrt{2\xi}$)

$$U(\rho) = \frac{e^2}{2} \left[-\frac{1}{r} + \frac{1}{r} \sum_{k=1}^{\infty} \zeta_H(2k, \frac{3}{2}) j_{2k}^s + \frac{\cos(2\mu \arctan \frac{\rho}{a}) - \cos(\pi\mu)}{2a\mu \sin \pi\mu} \right], \quad (18)$$

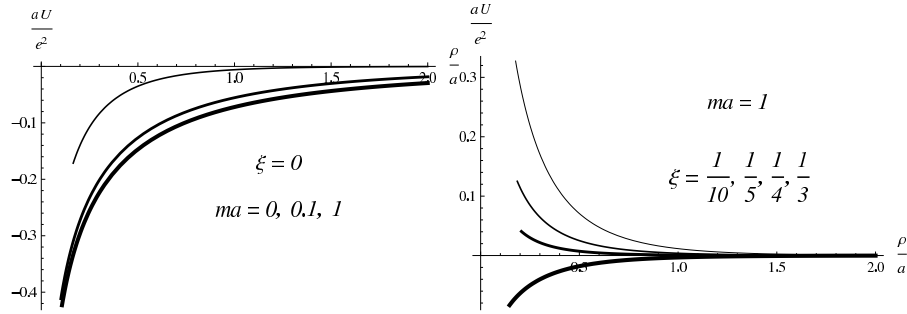


Fig. 2. The numerical simulation of the self-energy of a massive scalar field for $\xi = 0$ and for different parameters $ma = 0$ (thick line), $ma = 0.1$ (middle thickness) and $ma = 1$ (thin line). In the figure at right we show the numerical simulation for $ma = 1$ and for different parameters $\xi = 1/10$ (thick line) up to $\xi = 1/3$ (thin line).

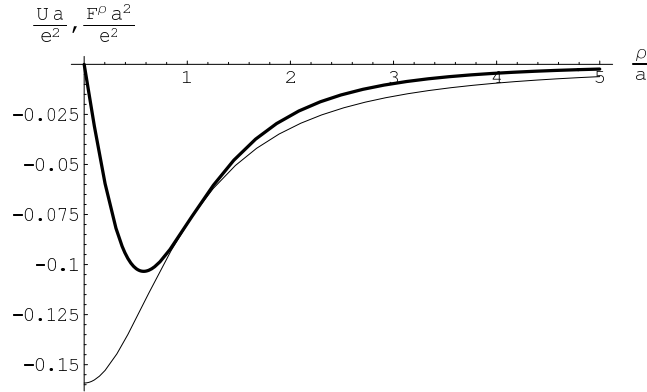


Fig. 3. Thin line is plot of potential and thick line is a plot of the self-force (17). The force has an extreme at point $\rho^* = a/\sqrt{3}$.

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where

$$j_2^s = -\zeta \frac{-1 + \dot{r}^2 + 2r\ddot{r}}{8},$$

$$j_4^s = \frac{3\zeta^2}{128} (\dot{r}^2 + 2r\ddot{r} - 1)^2 - \frac{r\zeta}{16} \left(2\ddot{r}\dot{r}^2 + 4rr^{(3)}\dot{r} + r \left(2\ddot{r}^2 + rr^{(4)} \right) \right).$$

As expected it is zero for $\xi = 1/8$ and it is divergent for $\xi = 1/2$. Far from the throat we obtain

$$U \approx -\frac{e^2}{2\rho^2} \frac{a\mu}{\tan \pi\mu}. \quad (19)$$

The numerical simulations are reproduced in Fig. 4.

For massive case the self-force is localized close to the throat of the wormhole.

3.3. General Profile

i) Electromagnetic field. To consider general profile of the throat let us perform the WKB analysis of radial equation with WKB parameter $\nu = l + 1/2$. We arrive with the following formula²⁶

$$U(\rho) = \frac{e^2}{2} \left[-\frac{1}{r} + \frac{1}{r} \sum_{k=1}^{\infty} \zeta_H(2k, \frac{3}{2}) j_{2k}^e + \frac{1}{a} \varphi_+^2(\rho) - \frac{1}{2a} \varphi_+^2(\rho) \frac{\varphi_+^2(\rho)}{\varphi_+^2(0)} \right], \quad (20)$$

where the solutions for zero mode read

$$\varphi_+^1 = 1, \quad \varphi_+^2 = \int_{\rho}^{\infty} \frac{a}{r^2} d\rho, \quad (21)$$

and

$$j_2^e = -\frac{-1 + \dot{r}^2 + 2r\ddot{r}}{8},$$

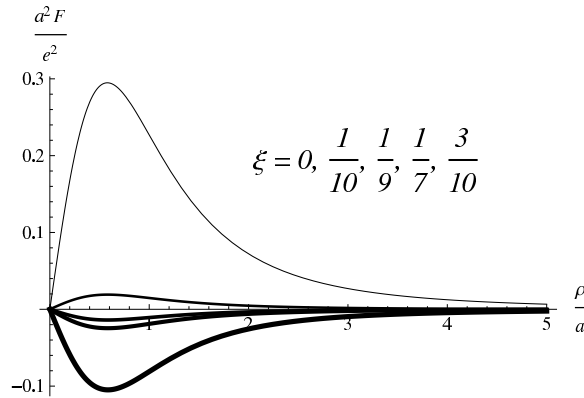


Fig. 4. The numerical simulation of the self-force on a massless scalar field for profile $r = \sqrt{\rho^2 + a^2}$ for different parameters from $\xi = 0$ (thick line) up to $\xi = \frac{3}{10}$ (thin line). For $\xi = \frac{1}{8}$ it is zero.

$$j_4^e = \frac{1}{128} [3 + 3\dot{r}^4 - 12r\ddot{r} - 4r^2\ddot{r}^2 - 2\dot{r}^2(3 + 2r\ddot{r}) - 32r^2\dot{r}r^{(3)} - 8r^3r^{(4)}].$$

Analysis of this expression for great distance from the throat of the wormhole gives the following expression for the self-potential

$$U = -\frac{e^2}{4\rho^2} \frac{a}{\varphi_+^2(0)} = -\frac{e^2}{4\rho^2} \left[\int_0^\infty \frac{d\rho}{r^2(\rho)} \right]^{-1}.$$

Note that it is always negative, hence the self-force is an attractive force. All information about the specific throat profile is encoded in the factor

$$\int_0^\infty \frac{d\rho}{r^2(\rho)}.$$

ii) Scalar field. Massless case. For arbitrary profile of the wormhole we have the following formula

$$U(\rho) = \frac{e^2}{2} \left[-\frac{1}{r} + \frac{1}{r} \sum_{k=1}^\infty \zeta_H(2k, \frac{3}{2}) j_{2k}(\rho, \rho) + g_0(\rho) \right], \quad (22)$$

with the same j_{2k}^s as above and

$$g_0 = -\frac{1}{A_+} \varphi_+^2(\rho) \varphi_+^1(\rho) + \frac{1}{2A_+} \left(\frac{\varphi_+^1}{\varphi_+^2} + \frac{\varphi_+^{\prime 1}}{\varphi_+^{\prime 2}} \right)_0 \varphi_+^2(\rho) \varphi_+^2(\rho), \quad (23)$$

where $A_+ = W_+(\varphi_+^1, \varphi_+^2) r^2(\rho)$. The functions $\varphi_+^{1,2}$ are the solutions of the equation

$$\varphi'' + \frac{2r'}{r} \varphi' - \xi R \varphi = 0. \quad (24)$$

Unfortunately, differently from the electromagnetic field case, there is no general solution of this equation for arbitrary ξ and r . Far from the throat we obtain the following expression

$$U \approx -\frac{e^2}{2\rho^2} A. \quad (25)$$

But we can not make any conclusion about sign of these expression because the constant A is expressed in terms of the zero mode which can not be found in closed form for arbitrary profile of the throat.

4. The Massive Wormhole Space-Time

The line element of massive wormhole has the following form^{23,24}

$$ds^2 = -e^{-\alpha(\rho)} dt^2 + e^{\alpha(\rho)} d\rho^2 + r^2(\rho) d\Omega^2, \quad (26)$$

where

$$r^2(\rho) = (\rho^2 + n^2 - m^2) e^{\alpha(\rho)}, \quad (27)$$

$$\alpha(\rho) = \frac{2m}{\sqrt{n^2 - m^2}} \left\{ \frac{\pi}{2} - \arctan \left(\frac{\rho}{\sqrt{n^2 - m^2}} \right) \right\}. \quad (28)$$

The radial coordinate ρ may be positive as well as negative, too. The square of the sphere of radial coordinate ρ , $S = 4\pi r^2(\rho)$, is minimized for $\rho = m$.

The renormalization procedure for the space-time with line element (26) is not so simple as it was for massless wormhole.^{26,25} The point is that the space-time under consideration has no ultrastatic form, that is $g_{tt} \neq 1$. For this reason the equation for A_t in static case does not coincide with that for scalar 3D Green function and we can not use the standard formulas for DeWitt-Schwinger expansion of the 3D Green function. The renormalization procedure for this case was developed in Ref. 28. The singular part of potential which has to be subtracted has the following form

$$A_{(t)}^{sing}(x^i; x^{i'}) = -e \left(\frac{1}{\sqrt{2\sigma}} + \frac{g_{t't'}, i' \sigma^{i'}}{4g_{t't'} \sqrt{2\sigma}} \right), \quad (29)$$

where σ is one-halfe of square of geodesic distance.

Taking into account this singular part we obtain the following form of the self-potential

$$U^{self} = -\frac{e^2}{\rho^2 + n^2 - m^2} \frac{me^{-\alpha}}{2 \tanh \pi b} \quad (30)$$

and tetrad component of the self-force

$$\mathcal{F}^{(\rho)} = -\partial_\rho U^{self} = \frac{e^2}{(\rho^2 + n^2 - m^2)^2} \frac{m(m - \rho)e^{-\alpha}}{\tanh \pi b}. \quad (31)$$

For massless wormhole, $m \rightarrow 0$, we recover results obtained in Ref. 26. Far from the wormhole's throat we obtain

$$\frac{U_{\rho \rightarrow +\infty}^{self}}{U_{\rho \rightarrow -\infty}^{self}} = \left| \frac{\mathcal{F}_{\rho \rightarrow +\infty}^{(\rho)}}{\mathcal{F}_{\rho \rightarrow -\infty}^{(\rho)}} \right| = e^{2\pi b}, \quad (32)$$

which is the consequence of the non symmetric form of the space-time under consideration. The numerical calculations are shown in Fig. 5.

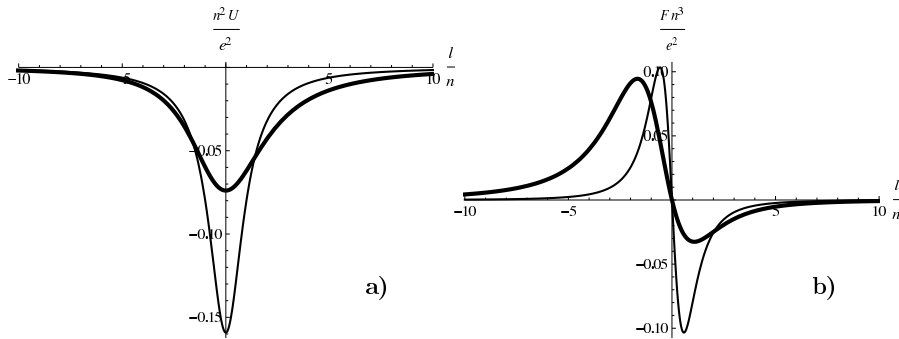


Fig. 5. The self-energy (a) and the self-force (b) for massless case (thin curves) and for massive wormhole case (thick curves) for $m/n = 0.7$. The zero value of the l corresponds to the sphere of the minimal square in both cases.

5. Conclusion

In above sections we considered the self-energy and self-force for particle with electric or scalar charges in space-time of the wormholes. The general conclusion for electrically charged particle is that they will be attracted to wormhole throat for arbitrary profile of the throat and arbitrary mass of the wormhole. The mass of the wormhole slightly changes the symmetric form of the self-force. This is due to the fact that the massive wormhole space-time has asymmetric form. From the astrophysical point of view it means that the wormhole's throat must be surrounded by particles. The self-energy of scalar particle exponentially falls down at the distance of the Compton wavelength of scalar field and falls down polynomial for massless field. The sign of the self-force depends on the non-minimal coupling of the scalar field.

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References

1. A. Einstein and N. Rosen, *Phys. Rev.*, **48** 73 (1935).
2. J. A. Wheeler, *Phys. Rev.*, **97**, 511 (1955).
3. M. S. Morris and K. S. Thorne, *Am. J. Phys.*, **56** 395 (1988).
4. M. S. Morris, K. S. Thorne and U. Yurtsever, *Phys. Rev. Lett.*, **61** 1446 (1988).
5. M. Visser, *Lorentzian Wormholes: From Einstein to Hawking*, (American Institute of Physics, Woodbury, NY, 1995).
6. S. Krasnikov, *Phys. Rev. D*, **62** 084028 (2000).
7. N. R. Khusnutdinov and S. V. Sushkov, *Phys. Rev. D*, **65** 084028 (2002); N. R. Khusnutdinov, *Phys. Rev. D*, **67** 124020 (2003).
8. R. Garattini, *Class. Quant. Grav.*, **22** 1105 (2005).
9. C. Armendariz-Picon, *Phys. Rev. D*, **65** 104010 (2002).
10. S. Sushkov, *Phys. Rev. D*, **71** 043520 (2005).
11. F. S. N. Lobo, *Phys. Rev. D*, **71** 084011 (2005).
12. F. S. N. Lobo, *Phys. Rev. D*, **71** 124022 (2005).
13. T. Damour, S. N. Solodukhin, *Phys. Rev. D*, **76** 024016 (2007).
14. R. Narayan, *New J. Phys.*, **7** 199 (2005).
15. N. S. Kardashev, I. D. Novikov and A. A. Shatskiy, *Int. J. Mod. Phys. D*, **16** 909 (2007).
16. B. S. DeWitt, R. W. Brehme, *Ann. Phys.*, **9** 220 (1960).
17. E. Poisson, *Living Reviews in Relativity*, **7** 6 (2004).
18. N. R. Khusnutdinov, *Physics - Uspekhi*, **48** 577 (2005)[*Uspekhi Fizicheskikh Nauk*, **175** 603 (2005)].
19. B. Linet, *Phys. Rev. D*, **33** 1833 (1986).
20. D. V. Gal'tsov, *Fortschr. Phys.*, **38** 945 (1990).
21. N. R. Khusnutdinov and V. B. Bezerra, *Phys. Rev. D*, **64** 083506 (2001).
22. A. R. Khabibullin, N. R. Khusnutdinov and S. V. Sushkov, *Class. Quantum Grav.*, **23** 627 (2006).

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- 23. K. A. Bronnikov, *Acta Phys. Pol. B*, **4** 251 (1973).
- 24. H. G. Ellis, *J. Math. Phys.*, **14** 104 (1973).
- 25. V. B. Bezerra and N. R. Khusnutdinov, *Phys. Rev. D*, **79** 064012 (2009).
- 26. N. R. Khusnutdinov and I. V. Bakhmatov, *Phys.Rev. D*, **76** 124015 (2007).
- 27. B. Linet, *Ann. Inst. Henri Poincare A*, **45** 249 (1986).
- 28. N. R. Khusnutdinov, A. A. Popov and L. N. Lipatova, *Class. Quantum Grav.*, **27** 215012 (2010).